Topic: Squeeze theorem, limit of an inequality

Question: Use the inequality to find the limit.

$$\lim_{x \to 3} f(x)$$

$$\text{if } x^2 \leq f(x) \leq x^3 - 6x$$

Answer choices:

A $\lim_{x \to 3} f(x) = 9$

B $\lim_{x \to 3} f(x) = 3$

C $\lim_{x \to 3} f(x) = 0$

D $\lim_{x \to 3} f(x) = \text{DNE}$

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Solution: A

We'll apply the limit to all three functions in the inequality.

$$x^2 \leq f(x) \leq x^3 - 6x$$

$$\lim_{x \to 3} x^2 \leq \lim_{x \to 3} f(x) \leq \lim_{x \to 3} x^3 - 6x$$

We'll solve the limits on either end.

$$(3)^2 \leq \lim_{x \to 3} f(x) \leq (3)^3 - 6(3)$$

$$9 \leq \lim_{x \to 3} f(x) \leq 27 - 18$$

$$9 \leq \lim_{x \to 3} f(x) \leq 9$$

In order to satisfy the inequality, the limit must be

$$\lim_{x \to 3} f(x) = 9$$

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Topic: Squeeze theorem, limit of an inequality

Question: Use the inequality to find the limit.

$$\lim_{x \to -4} f(x)$$

$$\text{if } x^2 - x + 3 \leq f(x) \leq x^2 - 2x - 1$$

Answer choices:

A $\lim_{x \to -4} f(x) = 15$

B $\lim_{x \to -4} f(x) = 23$

C $\lim_{x \to -4} f(x) = 7$

D $\lim_{x \to -4} f(x) = \text{DNE}$

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Solution: B

We'll apply the limit to all three functions in the inequality.

$$x^2 - x + 3 \leq f(x) \leq x^2 - 2x - 1$$

$$\lim_{x \to -4} x^2 - x + 3 \leq \lim_{x \to -4} f(x) \leq \lim_{x \to -4} x^2 - 2x - 1$$

We'll solve the limits on either end.

$$(-4)^2 - (-4) + 3 \leq \lim_{x \to -4} f(x) \leq (-4)^2 - 2(-4) - 1$$

$$16 + 4 + 3 \leq \lim_{x \to -4} f(x) \leq 16 + 8 - 1$$

$$23 \leq \lim_{x \to -4} f(x) \leq 23$$

In order to satisfy the inequality, the limit must be

$$\lim_{x \to -4} f(x) = 23$$

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Topic: Squeeze theorem, limit of an inequality

Question: Use the inequality to find the limit.

$$\lim_{x \to 5} f(x)$$

$$\text{if } 2x^2 - 4x - 5 \leq f(x) \leq x^2 + 2x - 10$$

Answer choices:

A $\lim_{x \to 5} f(x) = \text{DNE}$

B $\lim_{x \to 5} f(x) = 5$

C $\lim_{x \to 5} f(x) = 25$

D $\lim_{x \to 5} f(x) = 50$

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Solution: C

We'll apply the limit to all three functions in the inequality.

$$2x^2 - 4x - 5 \leq f(x) \leq x^2 + 2x - 10$$

$$\lim_{x \to 5} 2x^2 - 4x - 5 \leq \lim_{x \to 5} f(x) \leq \lim_{x \to 5} x^2 + 2x - 10$$

We'll solve the limits on either end.

$$2(5)^2 - 4(5) - 5 \leq \lim_{x \to 5} f(x) \leq (5)^2 + 2(5) - 10$$

$$50 - 20 - 5 \leq \lim_{x \to 5} f(x) \leq 25 + 10 - 10$$

$$25 \leq \lim_{x \to 5} f(x) \leq 25$$

In order to satisfy the inequality, the limit must be

$$\lim_{x \to 5} f(x) = 25$$

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